Session 4. Introduction to Modern Capture-Recapture Analysis: Open Populations.

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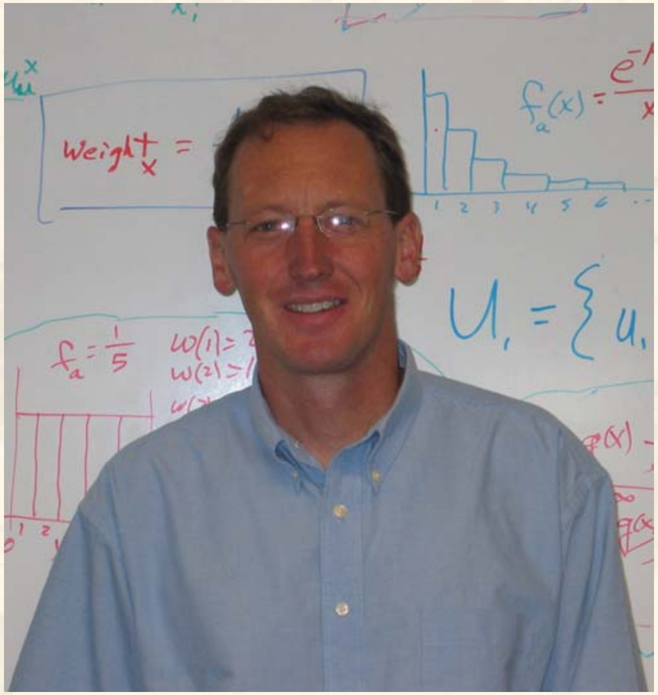

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Session 4. Introduction to Modern Capture-Recapture Analysis: Open Populations.

1. **For best service, close all other programs.**
2. Type questions in the form provided. We will attempt to answer some questions in the 2nd hour.
3. The seminar is being recorded in Windows Media format. If successful, the file will be available at www.west-inc.com later today or tomorrow.
4. Download course material at http://www.west-inc.com/webinars.php#downloads
5. E-mail other questions and comments to tmcdonald@west-inc.com.

To hear the seminar, telephone

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Open Populations 2

Topics for today:

1. Review
2. General model including covariates: CJS model
3. An Example
4. An Exercise

Review

- Session 3 covered several methods for open populations:
  - Jolly-Seber model
  - Manly-Parr method
  - Cormack-Jolly-Seber (CJS) model
- Session 4 (today):
  - Expand on CJS model, focusing on incorporating covariates

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Goals

At the end of today:

- You will have a better understanding of the CJS model.
- You will be able to fit a CJS model

Key contribution

- Jolly-Seber (unconditional) and Cormack-Jolly-Seber (conditioned on 1st capture) have been around since late 1960's
- (my hunch) Statistician's were incorporating covariates into CJS models informally during the late 1980's.

Key contribution

- Lebreton et al. (1992) formalized and provided examples where covariates were included in a CJS model
- Doing so allows testing of biological hypotheses.

Lebreton, J.-D., K. P. Burnham, J. Clobert, and D. R. Anderson. 1992. Modeling survival and testing biological hypotheses using marked animals: a unified approach with case studies, *Ecological Monographs* 62: 67-118.

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Other contributions

- McDonald and Amstrup (2001) proposed using the Horvitz-Thompson estimator to estimate population size from CJS models.
- Taylor et al. (2002) improved the HT variance estimator.

McDonald, T. L. and S. C. Amstrup. 2001. Estimation of population size using open capture-recapture models. *Journal of Agricultural, Biological, and Environmental Statistics* 6:206-220.

Taylor, M. K., J. Laake, H. D. Cluff, M. Ramsay, and F. Messier. 2002. Managing the risk from hunting for the Viscount Melville Sound polar bear population. *Ursus* 13:185-202.

CJS model: Intro

- Recall the definition of *covariate* from closed population modeling
  - *Type 1 covariates*: Known when animals are not seen
    - E.g., sex, age, effort, season, average weight, average distance from road, etc.
  - *Type 2 covariates:* Known *only* when an animal is captured
    - E.g., Habitat type, weight, distance from road, maternity status, family group/herd status, etc.

- Like Huggins model, the CJS approach currently incorporates *Type 1* covariates only.

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CJS model: Intro

- C., J., and S., conditioned on first capture, and wrote the likelihood of subsequent captures.
  - = Probability of obtaining the history of captures after the initial encounter *given* that the animal was encountered at all

CJS model: Intro

- Conditioning on 1st capture does the following:
  - Allows estimation of survival and capture probabilities from *any* (type of) sample of animals. I.e., not necessary to have a random sample of animals.
  - Allows us to ignore covariate values for animals that were never captured.
  - And, prohibits estimation of population size!

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CJS model: Intro

- BUT, we can estimate population size (at time *j*) if:
  - We assume every animal in the population at time *j* has positive capture probability.
  - We assume the model for capture probability of animals in population at time *j* is correct.

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CJS Model: Intro

- When estimating population size:
  - It is VERY nice to have a *random sample* of individuals in the population each trap occasion.

Polling question #1: Why?

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CJS model: Likelihood

- Assume we capture 2 animals with following histories:
  - Capture history for animal #1: 01010
  - Capture history for animal #2: 11100

CJS model: Likelihood

- Let p_{ij} = probability of capturing animal *i* on occasion *j*
- Let ϕ_{ij} = probability animal *i* survives from sampling occasion *j* to occasion *j+1*

- These are the CJS “real” parameters.

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CJS model: Likelihood

- Probability of obtaining history #1:
  - Pr(01010) = $\phi_{12}(1-p_{13})\phi_{13}p_{14}[(1-\phi_{14})+\phi_{14}(1-p_{15})]$

Animal could have died between occasion 4 and 5 OR survived and we failed to capture it.

$(1-\phi_{14})$ = probability animal died between occasion 4 and 5

$\phi_{14}(1-p_{15})$ = probability animal survived to occasion 5 times probability we failed to capture it.

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CJS model: Likelihood

- Probability of obtaining history #2:
  - Pr(11100) = $\phi_{21} p_{22} \phi_{22} p_{23} \left[(1-\phi_{23}) + \phi_{23}(1-p_{24})\left([1-\phi_{24}] + \phi_{24}[1-p_{25}]\right) \right]$

Pr(died in 3rd interval) → $(1-\phi_{23})$

P(survived 3rd interval and failed to be captured during 4th occasion) → $\phi_{23}(1-p_{24})$

P(not seen after 4th) → $[1-\phi_{24}] + \phi_{24}[1-p_{25}]$

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CJS model: Likelihood

- Writing likelihood this way gets complicated quick.
- We universally abbreviate as:
  - Pr(11100) = $\phi_{21} p_{22} \phi_{22} p_{23} \chi_{23}$
  - Where,

$$\chi_{ij} = (1 - \phi_{ij}) + \phi_{ij}(1 - p_{ij})\chi_{i(j+1)}$$

$$\chi_{ik} = 1$$

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CJS model: Likelihood

- Finally (assuming independence),
  - L = Pr(01010) Pr(11100)
  - ln(L) = ln(Pr(01010)) + ln(Pr(11100))

- Where are the covariates?

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CJS model: Likelihood

- *Reparameterize* by making the p's and ϕ's logistic functions of covariates.
- I.e.,:

$$p_{ij} = \frac{\exp(\beta_0 + \beta_1 x_{ij1} + \ldots + \beta_a x_{ija})}{1 + \exp(\beta_0 + \beta_1 x_{ij1} + \ldots + \beta_a x_{ija})}$$

$$\phi_{ij} = \frac{\exp(\gamma_0 + \gamma_1 z_{ij1} + \ldots + \gamma_b z_{ijb})}{1 + \exp(\gamma_0 + \gamma_1 z_{ij1} + \ldots + \gamma_b z_{ijb})}$$

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CJS model: Likelihood

- Note:
  - # covariates = *a* + *b*
  - Each covariate is a *matrix* of values (x_{ijk} or z_{ijk})

Histories

X_1

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{111} & x_{121} & x_{131} & x_{141} & x_{151} \\ x_{211} & x_{221} & x_{231} & x_{241} & x_{251} \end{bmatrix} \cdots$$

These do not count because can never estimate p_{i1} (first column of X_1: x_{111}, x_{211})

Z_b

$$\begin{bmatrix} z_{11b} & z_{12b} & z_{13b} & z_{14b} & z_{15b} \\ z_{21b} & z_{22b} & z_{23b} & z_{24b} & z_{25b} \end{bmatrix}$$

These do not count because cannot estimate ϕ_{ik} (last column of Z_b: z_{15b}, z_{25b})

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CJS model: Likelihood

- Now, likelihood is a function of the β's and γ's (and covariates).
- Only thing left to do is find the maximum.

CJS model: Size estimates

- How do we estimate population size?
  - Assume all animals have positive probability of capture, and that we have correctly modeled capture probabilities.

Polling question #2: If you magically knew that females entered your sample with probability 0.1 (10%), and you observed 2 females in the sample, how many females would you guess are in the population?

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CJS model: Size estimates

- Horvitz-Thompson estimator for population size at occasion *j* is,

$$\hat{N}_j = \sum_{i=1}^{n} \frac{h_{ij}}{\hat{p}_{ij}}$$

- Where,

  h_{ij} = the 0 or 1 capture indicator in capture history for animal i on occasion j.

  n = number of animals ever captured

- Errata: Errors in equation 5.5. Reverse "ji", and first sum should go to n (all animals), not n_j

CJS model: Size estimates

- How do we estimate variance of $\hat{N}_j$?

Trust me, you don't want to know. (use the delta method, see p. 241-245 of HCRA)

CJS model: Computation

- Conceivably, one could compute a CJS model in a spreadsheet using SOLVER.
  - But it is more complicated than Huggins model

- Can use MARK
  - No estimates of population size
- Can use MRA in R
  - Does what MARK does and estimates population size

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CJS model: Example

- Class experiment at TWS meeting:
  - Five capture occasions
  - Male volunteers flipped coin 10 times
    - "Captured" if 2 heads in a row (p_{male} = 0.25)
  - Female volunteers flipped coin 5 times
    - "Captured" if heads (p_{female} = 0.5)
  - Non-students drop out after 3rd occasion.
    - Permanent emigration
- There were 29 people that participated
  - 22 non-students
  - 7 students

Polling question #3 and #4: Have you used R and MARK before?

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CJS model: Exercise

- Reproduce estimates in Table 9.11 of HCRA.
  - Using MARK
    - “PIM approach” : See PDF containing section starting on p. 204 of HCRA. Data in Table 9.1 and files ‘DIPPER.INP’ and ‘DIPPER_TAB9-1.INP’.
    - “covariate approach” : Section starting on p. 214 is directly relevant to Table 9.11. MARK Design matrix Table 9.10B. Data in file ‘DIPPER_S.INP’

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CJS model: Exercise

– Using R (see p. 224)
  - dipper.RData is in course material (double click to open)
  - Explore covariates: x1 – x7, x1.sex – x7.sex, etc.
  - Attach the library: `library(mra)`
  - Try following statement: `tab911.fit <- F.cjs.estim( capture = ~ dipper.sex + x3 + x4 + x5 + x6 + x7 + x3.sex + x4.sex + x5.sex + x6.sex + x7.sex, survival = ~ dipper.sex + x2 + x3 + x4 + x5 + x6 + x2.sex + x3.sex + x4.sex + x5.sex + x6.sex, histories = dipper.histories)`
  - Fit other models (see "flood" model p. 229)
  - `plot(tab911.fit, type="s")`
  - `plot(tab911.fit)` and `plot(tab911.fit, occasions=1:6)`
  - `tab911.fit$s.hat; tab911.fit$p.hat; tab911.fit$n.hat`
  - `tab911.fit$se.s.hat, tab911.fit$se.n.hat`, etc.
  - `names(tab911.fit)`

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End