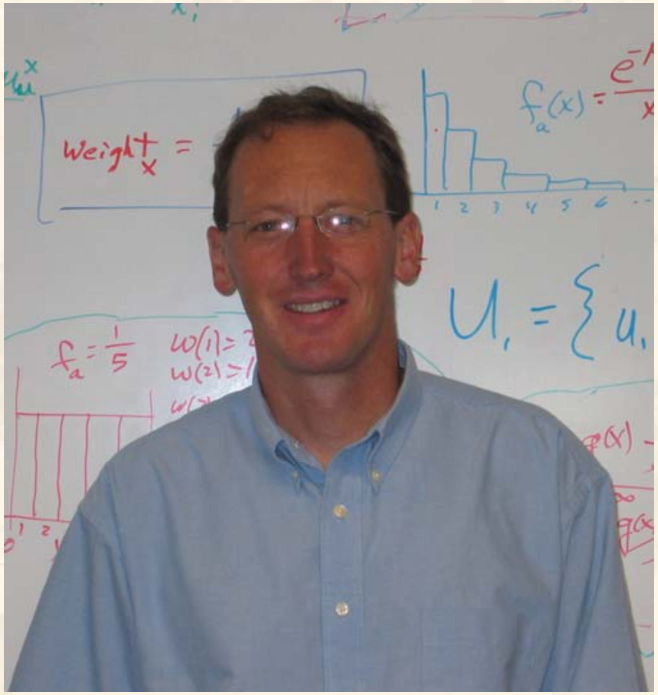
Session 2. Introduction to Modern Capture-Recapture Analysis: Closed Populations.

Dr. Trent McDonald

Western EcoSystems Technology, Inc.

Laramie and Cheyenne, Wyoming



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Session 2. Introduction to Modern Capture-Recapture Analysis: Closed Populations.

1. **For best service, close all other programs.**
2. Type questions in the form provided. We will attempt to answer some questions in the 2nd hour.
3. Minimize the form in your upper left corner using the tab on the left hand side of the box.
4. The seminar is being recorded in Windows Media format. If successful, the file will be available at www.west-inc.com next week.
5. E-mail other questions and comments to tmcdonald@west-inc.com.

To hear the seminar, telephone

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Closed Populations 2

1. Brief review
2. General model including covariates: Huggins model
3. Exercises

Polling question #1: My level of experience with capture-recapture analyses is…

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Review

- Lincoln-Peterson and Schnabel Census were useful early estimators for specific models of Otis et al.(M_0 and M_t)
- Models of Otis et al. incorporate:
  - Time variation
  - Behavioral effects
  - General heterogeneity (unspecified)
  - And combinations

Review

- Many of the Otis et al. models have estimators (see Table 4.1, p. 73 of HCRA)
- But, none (?) of these estimators incorporate variables specific to individuals or time.
- They therefore model “generic” variation.
- Sometimes you know what causes variation in capture probabilities.

Key Contribution

- Several people realized including covariates was a good idea and proposed CR models that allowed capture probability to depend on certain types of exogenous covariates
- BUT, Huggins (1989, 1991) realized we could condition on the set of captured animals and avoid needing covariates on all (seen and unseen) animals

Huggins, R. M. 1989. On the statistical analysis of capture experiments. *Biometrika* 76:133-140

Huggins, R. M. 1991. Some practical aspects of a conditional likelihood approach to capture experiments. *Biometrics* 47:725-732.

Key Contribution

- Turns out, all the Otis et al. models (and more) can be fitted using the Huggins model via judicious choice of covariates.

Huggins model: Intro

- Def'n: *Covariates* = individual, environmental, auxiliary, or concomitant variables recorded or measured during the course of a capture-recapture experiment
- Examples
  - – Individual: *Sex, Age, Weight*
  - – Environmental: *Weather, Habitat type*
  - – Concomitant: *Distance from road, Season, Effort*

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Huggins model : Intro

- *Type 1 covariates*: Known for captured animals, even when they are not seen
  - E.g., sex, age, effort, season, average weight, average distance from road, etc.
- *Type 2 covariates:* Known *only* when an animal is captured
  - E.g., Habitat type, weight, distance from road, maternity status, family group/herd status, etc.

Huggins model : Intro

- Huggins model allows incorporation of *Type 1* covariates only.
- Some current research in CR is attempting to incorporate *Type 2* covariates.
  - While these models would be amazingly useful, they are not ready for us to use…yet.

Huggins model: Overview

- Huggins did the following:
  - separated initial capture probabilities from subsequent capture probabilities
  - wrote a *conditional* likelihood as a function of initial and subsequent capture probabilities
  - related parameters to covariates via logistic regression equations

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Huggins model: Likelihood

- Assume we capture 2 animals with following histories:
  - Capture history for animal #1: 111
  - Capture history for animal #2: 010

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Huggins model: Likelihood

- To define likelihood, we need two quantities:
  - Probability of observing the history
  - Probability of observing the animal
- From these, we calculate probability of observing the history *given* that we observed the animal.

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Huggins model: Likelihood

- Let p_{ij} = probability of initial capture for animal i on occasion j
- Let c_{ij} = probability of recapture for animal i on occasion j
- These are the Huggin's model "real" parameters.

Polling question #2: What is probability of recapture (c_{ij}) prior to an individual's initial capture?

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Huggins model: Likelihood

- Probability of obtaining our histories:
  - #1 : Pr(111) = $p_{11}c_{12}c_{13}$
  - #2 : Pr(010) = $(1-p_{21})p_{22}(1-c_{23})$
- Probability of observing the animals:
  - #1 : Pr(capture #1) = $1-(1-p_{11})(1-p_{12})(1-p_{13})$
  - #2 : Pr(capture #2) = $1-(1-p_{21})(1-p_{22})(1-p_{23})$

Why? →

All possible histories: 111, 110, 101, 100, 011, 010, 001, 000

Sum of all history probabilities = 1

Observe animal under all histories except 000.

Pr(observe animal) = 1 – Pr(000)

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Huggins model: Likelihood

- Probability of history *given* animal was observed = Pr(history|animal observed) = Pr(history)/Pr(animal observed)
  - #1: $L(1) = \dfrac{p_{11}c_{12}c_{13}}{1-(1-p_{11})(1-p_{12})(1-p_{13})}$
  - #2: $L(2) = \dfrac{(1-p_{21})p_{22}(1-c_{23})}{1-(1-p_{21})(1-p_{22})(1-p_{23})}$

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Huggins model: Likelihood

- Likelihood of entire data set is:
  - $L = L(1)L(2)$ ← Why?

We assume animals are independent

- Log(likelihood) is:
  - $\ln(L) = \ln(L(1)) + \ln(L(2))$ ← Why do we take ln()?

- Maximum of ln(L) is at same place as maximum of L.
- ln(L) is linear, whereas L is multiplicative, and linear equations are easier to maximize.

Huggins model: Likelihood

- Log likelihood, ln(L), is a function of p's and c's
- Where are the covariates?

Huggins model: Likelihood

- Huggins *reparameterized* and made the p's and c's a logistic function of covariates.
- I.e.,:

$$p_{ij} = \frac{\exp(\beta_0 + \beta_1 x_{ij1} + \ldots + \beta_a x_{ija})}{1 + \exp(\beta_0 + \beta_1 x_{ij1} + \ldots + \beta_a x_{ija})}$$

$$c_{ij} = \frac{\exp(\gamma_0 + \gamma_1 z_{ij1} + \ldots + \gamma_b z_{ijb})}{1 + \exp(\gamma_0 + \gamma_1 z_{ij1} + \ldots + \gamma_b z_{ijb})}$$

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Huggins model: Likelihood

- Note:
  - # covariates = *a* + *b*
  - Each covariate is a *matrix* of values (x_{ijk} or z_{ijk})

Histories $\qquad X_1 \qquad Z_b$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{111} & x_{121} & x_{131} \\ x_{211} & x_{221} & x_{231} \end{bmatrix} \cdots \begin{bmatrix} z_{11b} & z_{12b} & z_{13b} \\ z_{21b} & z_{22b} & z_{23b} \end{bmatrix}$$

Polling question #3: In regular regression, where Y is a vector, what is the covariate X_1? A constant, a scalar (single value), a vector, or a matrix?

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Huggins model: Likelihood

- Now, likelihood is a function of the β's and γ's (and covariates).
- Only thing left to do is find the maximum.
  - I.e., find values of β's and γ's that maximize ln(L).
- This involves Newton-Raphson or some other numerical method: Details omitted.

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Huggins model: Likelihood

- How do we estimate variance? (of β's and γ's, and of p's and c's)
  - Variance estimates are based on curvature (2nd derivative) of the likelihood surface near the maximum.
  - Heuristic: upside down bowl has less variance in maximum than upside down plate.

Trust me, you don't want to know the details behind why this is a good thing to do.

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Huggins model: Computation

- The "Huggins spreadsheet"
  - Inspection of "Hugginsspreadsheet V3.xls"

- Once calculations are understood, Huggins model could be programmed in SAS or R or C or Fortran, etc.

Polling question #4: At this point, I …

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Huggins model: Computation

- Wimps can use MARK.
  - Demonstration of same calculations in MARK.
    - Mb, Mh, Mbh

Thank you

- Polling question #5: What is the sample size for closed population models?

- Response to questions